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AN APPROACH TO THE PROBLEM OF ESTIMATING SEVERE AND REPEATED GUST LOADS FOR MISSILE OPERATIONS

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SUMMARY

An analysis of available airplane measurements of vertical gust velocity is presented in order to arrive at a simple description of the frequency and intensity of gust velocities experienced by airplanes in operations. For the purpose of application to missile operations, the results obtained are modified to eliminate the effects of storm-avoidance procedures normally followed in airplane operations. The frequency distributions of gust velocity are then converted to a form appropriate for use in power spectral response calculations. Methods of applying the results to the estimation of the large and the small repeated loads in missile operations are then developed. Simple methods of estimating the gust loadings that will be exceeded with a given probability are presented in terms of missile response parameters and turbulence parameters. The limitations of the present results are also discussed briefly.

INTRODUCTION

The effects of atmospheric turbulence on airplane structural loads have been of concern for many years. Recently, it has become increasingly clear that certain types of missiles and unmanned vehicles are also sensitive to turbulence in regard to structural loading and control problems. It is the purpose of the present paper to extend recent results on the estimation of gust loads for airplane operations (refs. 1 and 2) to the case of missile operations. In reference 1, initial descriptions of the frequency and intensity of atmospheric turbulence and their variation with altitude were derived in terms of discrete or derived gust velocities, and methods of applying these data to load calculations for airplane operations were presented. More recently the development of random-process applications to gust response problems has, in turn, led to efforts to utilize these data in order to establish an appropriate description of the turbulence environment and a procedure for response calculations in terms of the power spectra of turbulence (ref. 2). This procedure provides a more realistic representation of the turbulence field and furthermore is more suitable to the treatment of missile stability and elastic dynamics.

In the present paper, use is made of data on atmospheric turbulence obtained from airplane operations. The application of airplane gust data to the calculation of gust loads on missiles involves a number of problems among which the following two are of importance. First, modifications to the atmospheric-environment data obtained from airplane surveys are required in order to account for the effects of the storm-avoidance procedures normally followed in airplane operations and not applicable to missile operations. Second, and perhaps a more serious problem, is that concerning the flight-path angle of the missile. For flight paths that are moderately inclined to the horizontal, the indications of the approximate isotropy of atmospheric turbulence (refs. 3 and 4) suggest that the airplane data would apply reasonably well. For flight paths that are more near vertical, however, serious questions exist as to the applicability of gust data obtained from airplanes in horizontal flight. However, no adequate alternative appears currently available for this vertical-flight case. Thus, the present study might be considered to apply best to missile operations in flight paths similar to those of airplanes or in moderately inclined flight paths and to apply only in a very crude way to near-vertical flight paths.

This paper presents the results obtained from an examination of available data on the frequency and intensity of atmospheric gust velocities and their variation with altitude and, in this respect, brings up to date the results reported earlier in reference 1. These data are then adjusted for the present purpose of missile application to account for the effects of airplane storm-avoidance practices. The distributions of gust velocities are then converted into a form appropriate for use in power spectral response calculations in accordance with methods of reference 2. Methods of applying these results to the calculation of both the large and the smaller repeated gust loads in missile operations are then developed.

SYMBOLS

$\bar{A}$ gust-response factor, σ_y/σ_w

a_n airplane vertical acceleration, g units

b scale parameter in probability distribution of root-mean-square gust velocity

$\bar{C}$ gust-response factor, $\dfrac{K_g \rho_o m S V_e}{2W}$

$\bar{c}$ mean geometric chord, ft

$\bar{D}(y)$ average flight miles required to exceed given values of response quantity y

D_r flight distance in gust-critical flight segment, miles

E_1, E_2 functions of flight distance and P_{ex} (defined in equations that follow equations (29) and (30), respectively)

$\hat{F}(\sigma_w)$ cumulative probability distribution of root-mean-square gust velocity

$\hat{f}(\sigma_w)$ probability density distribution of root-mean-square gust velocity

$\bar{G}(\ )$ average number of peaks of specified response per mile of flight exceeding given values of argument

G_o average number of peaks of specified response per mile of flight

g acceleration due to gravity

$H(\Omega)$ frequency-response function

$\sqrt{\frac{I(K,s)}{\pi}}$ gust-response factor

K airplane mass ratio, $\frac{4W}{g\pi\rho S\bar{c}}$

K_g gust-response factor (ref. 5)

k turbulence intensity factor describing variations with altitude

L scale of turbulence, ft

m slope of lift curve per radian

N_o average number of peaks of specified response per second of flight

P proportion of total flight time or distance in turbulence

$P_{ex}(\ )$	probability of exceeding specified value of argument
S	wing area, sq ft
$s = \bar{c}/L$	
U_{de}	derived gust velocity, fps
V	true airspeed, fps
V_e	equivalent airspeed, $V\sqrt{\rho/\rho_o}$
W	airplane or missile weight, lb
y	response quantity
y_L	specified value of a response quantity
μ_g	airplane mass parameter, $\frac{2W}{m\rho\bar{c}gS}$
ρ	air density, slugs/cu ft
ρ_o	air density at sea level, slugs/cu ft
σ_{an}	root-mean-square normal acceleration
σ_w	root-mean-square gust velocity
σ_y	root-mean-square response y
$\Phi(\Omega)$	power-spectral-density function
Ω	frequency, radians/foot

Subscripts:

1	nonstorm turbulence
2	storm turbulence

GENERAL APPROACH

Turbulence Model

The approach to be followed in the present study is basically that utilized in references 1 and 2. In reference 1, a simplified model was used to describe the turbulence experienced in normal airplane operations. In essence, this model assumed that the turbulence experienced in normal operations could be broadly considered to be of two general types: one consisting of a severe turbulence condition, represented by turbulence encountered in thunderstorms, and termed "storm" turbulence and the other consisting of a considerably less severe condition, perhaps representative of conditions in moderately rough clear air, and termed "nonstorm" turbulence. The turbulence for these two conditions was described by appropriate average frequency distributions which defined the average number of gusts per mile exceeding given values of derived gust velocity U_{de}. On this basis, the turbulence for a given operation or set of atmospheric conditions may be viewed as being given by the following relation:

$$\overline{G}(U_{de}) = P_1\overline{G}_1(U_{de}) + P_2\overline{G}_2(U_{de}) \tag{1}$$

where

$\overline{G}(U_{de})$ overall frequency distribution of U_{de} encountered in a given operation or part of an operation and normally given in terms of the average number of gusts per mile of flight exceeding a given value

$\overline{G}_1(U_{de})$ frequency distribution of U_{de} for nonstorm turbulence

$\overline{G}_2(U_{de})$ frequency distribution of U_{de} for storm turbulence

P_1, P_2 proportion of total flight distance in nonstorm and storm turbulence, respectively

The appropriate values of P_1 and P_2 and the appropriate distributions of $\overline{G}_1(U_{de})$ and $\overline{G}_2(U_{de})$ can conceivably vary with atmospheric conditions. Some of the parameters which could affect these quantities are

Altitude
Latitude
Surface conditions (land, water, smooth or rugged terrain)
Seasons of the year
Route of operation

Although efforts to evaluate the variations in turbulence frequency and intensity have been made for each of these parameters, no large and persistent differences have as yet been established for any of these parameters except altitude. For this parameter certain trends appear well established, as indicated in a subsequent section. For the other parameters the lack of any clear patterns has been, in part, a consequence of the limitations in the available data which are mostly confined to operation within and close to the United States. (See, for example, refs. 5 and 6.) Also, in many cases the records covered a variety of operating conditions in regard to locale, latitude, and even seasons of the year, and no separation of the data was possible. In several investigations, direct comparisons of turbulence experienced at different seasons and on different routes were made and indicated that some differences were present. However, the differences observed were neither large nor consistent and thus appeared of secondary importance.

As a consequence of the foregoing limitations in the data, the current information on turbulence is restricted to variations in the overall turbulence pattern with altitude. Estimates of the quantities P_1, P_2, $\overline{G}_1(U_{de})$, and $\overline{G}_2(U_{de})$ and their variation with altitude were given in reference 1 for use in transport-type operations. These estimates were based on the limited data available at that time. Since that time, a large amount of additional data has been collected, particularly for flight altitudes above 10,000 feet and up to altitudes of 55,000 feet. Also, the data on thunderstorms have since been examined in greater detail in reference 7.

For the foregoing reasons, it appeared appropriate first to revise the estimates given earlier in reference 1 for airplane operations. In addition, for the present purpose of missile application, adjustments to these results are required to account for the effects of storm-avoidance procedures normally followed in the airplane operations from which the gust data were obtained.

Power Spectral Representation

The description of the turbulence in terms of distributions of derived gust velocities, as given by equation (1), is then converted into a form appropriate for use in power spectral response calculations in accordance with the general methods outlined in reference 2. This conversion provides a turbulence description in terms of the probability distributions of the root-mean-square gust velocities. The turbulence representation obtained for given flight operations in this manner can be expressed in a form analogous to that given by equation (1) as

$$\hat{f}(\sigma_w) = P_1\hat{f}_1(\sigma_w) + P_2\hat{f}_2(\sigma_w) \tag{2}$$

where

$\hat{f}(\sigma_w)$ probability density distribution of root-mean-square gust velocity

$\hat{f}_1(\sigma_w)$ probability density distribution of root-mean-square gust velocity for nonstorm turbulence

$\hat{f}_2(\sigma_w)$ probability density distribution of root-mean-square gust velocity for storm turbulence

As in equation (1), P_1 and P_2 represent the proportion of total flight time spent in nonstorm turbulence and in storm turbulence, respectively. This conversion is performed on the basis of an assumed power spectral shape as in reference 2.

Gust Response Calculations

The representation of the turbulence environment in the form of equation (2) can then be applied to the problems of gust response calculations by utilizing the general methods described in reference 2. As indicated therein, for given conditions the expected response history in y of the airplane (where y may be taken as the airplane acceleration, bending moment, stress, or any response quantity) is given by

$$\overline{G}(y) = G_o \int_0^\infty \hat{f}(\sigma_w)\, e^{-y^2/2\sigma_w^2\overline{A}^2}\, d\sigma_w \tag{3}$$

where

$\overline{G}(y)$ average number of response peaks per mile of flight exceeding given values of y

G_o average number of response peaks per mile of flight in rough air

$\bar{A}$ ratio of root-mean-square values of specified response y and vertical gust velocity (for a given airplane and set of conditions and within the framework of linear theory, this value depends only upon the form of the gust spectrum), σ_y/σ_w

In the present study, the choice of appropriate functional forms for the gust distribution $\bar{G}(U_{de})$ yields a simple form for the probability density distribution of root-mean-square gust velocity $\hat{f}(\sigma_w)$, which, in turn, permits a closed-form integration of equation (3) that yields a number of results that permit rapid estimation of the large and the repeated gust loads.

TURBULENCE ENVIRONMENT

In this section, flight measurements of atmospheric turbulence are reviewed and a description of the turbulence environment is derived in terms of the quantities defined in equation (1) $\left(P_1,\ P_2,\ \bar{G}_1(U_{de}),\right.$ and $\left.\bar{G}_2(U_{de})\right)$ and in equation (2) $\left(\hat{f}_1(\sigma_w)\right.$ and $\left.\hat{f}_2(\sigma_w)\right)$.

Nonstorm and Storm Gust Distributions $\bar{G}_1(U_{de})$ and $\bar{G}_2(U_{de})$

Flight measurements of the gust-velocity distributions indicate that the nonstorm and storm gust distributions $\bar{G}_1(U_{de})$ and $\bar{G}_2(U_{de})$, respectively, vary widely from one day or condition to the next. They do, however, on the average show persistent trends with altitude. In reference 1, two basic distributions, herein designated by $\bar{G}_1{}^*(U_{de})$ and $\bar{G}_2{}^*(U_{de})$, were chosen on the basis of the data available at that time and estimates were then made of their variation with altitude. In these terms, the distributions $\bar{G}_1(U_{de})$ and $\bar{G}_2(U_{de})$ for a given altitude are given by

$$\bar{G}_i(U_{de}) = \bar{G}^*(U_{de}/k_i) \qquad (i = 1,\ 2) \qquad (4)$$

where the quantity k is an intensity parameter which varies with altitude. The basic distributions $\bar{G}_1{}^*(U_{de})$ and $\bar{G}_2{}^*(U_{de})$ used in reference 1 are given in figure 1. The variations in k for the two types of turbulence are designated by k_1 and k_2 and the results used in reference 1 for the variations with altitude of these two quantities are shown in figure 2. Note that for the storm-turbulence case, the intensity was taken as the same at all altitudes $\left(k_2 = 1.0\right)$.

As a part of the present study, a review was made of the more recent data on the variations of turbulence with altitude. This review indicated that a minor modification in the choice of $\overline{G}_1{}^*\left(U_{de}\right)$ was desirable in order to reflect more closely the values of $\overline{G}_1(0)$ (subsequently designated as G_0) measured in flight tests. The modified distribution $\overline{G}_1{}^*\left(U_{de}\right)$ is shown in figure 1 and is given by

$$\overline{G}_1{}^*\left(U_{de}\right) = 20e^{-U_{de}/2.2} \tag{5}$$

The estimates of k_1 given in reference 1 were, however, retained unchanged except that estimates for the lower altitudes (0 to 5,000 feet) were added and are shown in figure 2. This extension was made in order to represent more adequately conditions at very low altitudes which appear of particular interest in certain applications.

In regard to the distributions of storm turbulence $\overline{G}_2\left(U_{de}\right)$, it appeared appropriate to modify the distributions utilized in reference 1, as indicated in figure 1, in order to reflect more closely the results obtained in reference 7. The curve shown is based on the results given in table III of reference 7 and represents a more severe turbulence condition than that given in reference 1. In addition, this modification has the additional advantage for present purposes of yielding a simple exponential form for the distributions of $\overline{G}_2{}^*\left(U_{de}\right)$ (as can be seen from the straight-line character of the curve on semilogarithmic paper). The distribution is given by

$$\overline{G}_2{}^*\left(U_{de}\right) = 15e^{-U_{de}/5.3} \tag{6}$$

The more severe turbulence condition represented by the present choice is, by itself, not significant inasmuch as the storm turbulence that applies to operations depends also on the values for P_2.

In addition, the results of reference 7 suggest that for altitudes above 20,000 feet the intensity of the turbulence decreases with increase in altitude. This result is in accord with the general impression of many pilots and is consistent with what may be expected from meteorological considerations. (The relatively low moisture content and greater stability of the atmosphere at the higher altitudes would tend to make smaller amounts of energy available for vertical and turbulent motion.)

As a consequence, it appeared reasonable to allow k_2 to decrease with altitude above 25,000 feet, as indicated in figure 2. The choice made, however, is arbitrary.

By combining the results of figures 1 and 2 in accordance with equation (4), the distributions $\bar{G}_1(U_{de})$ and $\bar{G}_2(U_{de})$ appropriate for each of the altitude brackets are obtained and are shown in figure 3. For the lowest 10,000 feet, separate distributions are shown for the altitude brackets of 0 to 2,000 feet and 2,000 to 10,000 feet. These frequency distributions are given as follows:

$$\bar{G}_1(U_{de}) = 20e^{-U_{de}/2.2k_1} \tag{7}$$

and

$$\bar{G}_2(U_{de}) = 15e^{-U_{de}/5.3k_2} \tag{8}$$

where the values of k_1 and k_2 for the various altitudes are defined in figure 2.

It is of interest to note that the coefficients (to be designated by G_0) on the right-hand sides of equations (5) to (8) - namely, 15 and 20 - define the average number of gust peaks per mile. Except for the difference in units, this quantity is approximately related to the characteristic frequency N_o of reference 2 (the number of positive acceleration peaks per second). These definitions imply that

$$N_o \approx \frac{V}{(2)(1.467)(3600)} G_0 \approx \frac{V}{10560} G_0 \tag{9}$$

where V is the airplane speed in feet per second, and the coefficient $\frac{1}{2}$ arises from the fact that N_o is based on positive peaks only, whereas G_0 and the gust data include both positive and negative peaks. A characteristic value for the airspeed V for the airplanes used in the gust-data collections is about 350 feet per second which yields values of N_o of about 0.5 and 0.7. These values are reasonably consistent with the estimates of N_o given in reference 2 for most of the airplanes considered therein, and, thus, the relation of equation (9) is assumed to apply in subsequent considerations.

The foregoing estimates of the gust distribution were, in large part, based on center-of-gravity normal-acceleration measurements obtained from transport operations. It is well known that for many airplanes the effects of airplane flexibility give rise to substantial amplifications of the airplane center-of-gravity accelerations. As a consequence, the values of U_{de} derived from such amplified accelerations would likewise tend to be amplified. In reference 1, a simple correction or reduction of 20 percent was applied to the acceleration measurements and thus to the gust velocities to account for this effect. In the present investigation, the same correction was used in the determination of the distribution $\overline{G}_1(U_{de})$. However, for the distribution $\overline{G}_2(U_{de})$, no such correction was believed necessary inasmuch as the airplanes used in obtaining most of the thunderstorm gust data were relatively stiff and dynamic effects on the center-of-gravity accelerations were small. In comparing these distributions with operational data, this difference must be kept in mind and the effects of flexibility on the operational data be considered.

Proportions of Flight Distance in Nonstorm and Storm Turbulence P_1 and P_2

In order to determine appropriate proportions of flight distance in nonstorm and storm turbulence P_1 and P_2 for transport operations, equation (1) was used with the results of figure 3 to approximate the gust distributions measured in transport operations. Simple graphical procedures were used and yielded estimates of P_1 and P_2 which gave good representations of the measured data. Inasmuch as the data from various operations for a given altitude bracket varied widely, average values of P_1 and P_2 were obtained. The values of P_1 and P_2 obtained for the various altitudes are shown in figure 4. For comparison, the values of P_1 and P_2 from reference 1 are also shown. The same 20-percent correction, discussed previously, to account for dynamic effects was also applied to the operational gust data in deriving estimates of P_1 and P_2.

Inasmuch as the operational data available for the higher altitudes (above 20,000 feet) were limited, estimates of P_1 and particularly of P_2 are at best crude. In estimating values of P_2, no flight data were available and recourse to indirect evidence such as that given in reference 8 on the distribution of thunderstorm cloud tops was necessary. These data were used as a basis for extrapolating the values of P_2 obtained from the gust data for the lower altitudes to the higher altitudes.

The present values of P_1 and P_2 shown in figure 4 differ in a number of respects from those given in reference 1. In regard to the values of P_1, the most significant difference is the increase in value for the altitudes between 20,000 and 40,000 feet. This increase is indicated by recent unpublished studies and in reference 9 and is associated with the presence of the jet stream in this altitude range. The decrease in values of P_2 from those given in reference 1 also appears large. However, the distribution $\overline{G}_2(U_{de})$ has, for present purposes, been selected to be more severe than that used in reference 1. The net effect of these two modifications is a small increase in the severe turbulence condition for the present estimates.

Overall Gust Distribution $\overline{G}(U_{de})$

By combining the results obtained in figures 3 and 4 in accordance with equation (1), the overall distributions of gust velocity $\overline{G}(U_{de})$ for the various altitude brackets are obtained and are given in figure 5. For this purpose, average values of P_1 and P_2 for the various altitude brackets were determined from figure 4. The actual values used are summarized in the following table:

Altitude, ft	P_1	P_2
0 to 2,000 . . .	0.32	0.00025
2,000 to 10,000 . . .	0.08	0.0008
10,000 to 20,000 . . .	0.045	0.0004
20,000 to 30,000 . . .	0.06	0.00013
30,000 to 40,000 . . .	0.065	0.000045
40,000 to 50,000 . . .	0.023	0.00001
50,000 to 60,000 . . .	0.02	0

The frequency distributions of figure 5 are all given by the following expression:

$$\overline{G}(U_{de}) = 20P_1 e^{-U_{de}/2.2k_1} + 15P_2 e^{-U_{de}/5.3k_2} \tag{10}$$

where the values of P_1 and P_2 are those given in the foregoing table and the values of k_1 and k_2 are obtained from curves in figure 2 at the midpoints of the various altitude brackets.

Modifications To Account for Storm-Avoidance Procedures

For purposes of missile applications, modifications are required to the foregoing results in order to eliminate the effects on the data of airplane storm-avoidance procedures. These modifications can at best be only crudely estimated on the basis of available information. Available information indicates that little effective effort is normally made by pilots to avoid the lighter or nonstorm-turbulence areas. However, serious and more effective efforts are normally made to avoid storm-turbulence areas. Little quantitative information is available on the consequences on the gust experience of such storm-avoidance procedures. Some indirect information that has some bearing on this problem is, however, available and includes data on the frequency of thunderstorms, their average horizontal dimensions and time durations, and their altitude extent. Roughly it is estimated that thunderstorms occur, on the average, on about 30 days per year for the United States and have an average duration of perhaps two hours. It would thus appear that for a given location the probability of a thunderstorm being present is approximately equal to $\frac{(30)(2)}{(360)(24)}$ or 0.007. Comparison of this value with those of figure 4 for airplane operations suggests that airplanes may well avoid a large part of the atmospheric storms. Inasmuch as thunderstorms are probably less frequent on a worldwide basis, somewhat lower values than 0.007 appeared appropriate for present purposes. The values of P_2 selected as representative for missiles in all-weather operations are those shown in figure 4(b).

Application of these modified values of P_2 in equation (10) yields the distribution $\overline{G}(U_{de})$ appropriate for all-weather missile operations, and these distributions are given in figure 6. In general, they represent a more severe gust history than that given earlier for airplane operations and for the less frequent gusts, say, $\overline{G}(U_{de}) = 10^{-5}$, are roughly 40 percent more severe at the various altitude levels. Analytically these distributions may be represented by the same expression as given earlier in equation (10).

Conversion to Power Spectral Form

The distribution $\overline{G}(U_{de})$ in figure 6 may be converted into a form appropriate for power spectral response calculations by making use of the approach of reference 2. As indicated therein, if the power spectral form of the turbulence is assumed invariant, the turbulence history experienced by an airplane may be defined by the probability density distribution of the root-mean-square gust velocity $\hat{f}(\sigma_w)$. On the basis of the results of reference 2 and as given in equations (3) and (9) herein, $\hat{f}(\sigma_w)$ is related to the airplane acceleration history $\overline{G}(a_n)$, in terms of the average number of acceleration peaks per mile exceeding given values of a_n, by the relation

$$\overline{G}(a_n) = \frac{10560}{V} N_o \int_0^{\infty} \hat{f}(\sigma_w) e^{-a_n^2/2\sigma_w^2 \overline{A}^2} d\sigma_w \tag{11}$$

where

N_o characteristic frequency of airplane acceleration response and approximately specifies average number of peak accelerations per second

$\overline{A} = \sigma_{an}/\sigma_w$ for the specified airplane and spectral form

Inasmuch as

$$U_{de} = \frac{2W}{K_g \rho_o m S V_e} a_n = \frac{1}{\overline{C}} a_n \tag{12}$$

where $\overline{C} = \frac{K_g \rho_o m S V_e}{2W}$, the derived gust velocity may be viewed as a reduced or normalized acceleration and the distribution of peak values of U_{de} is, in turn, from equations (11) and (12) given by

$$\overline{G}(U_{de}) = \frac{10560}{V} N_o \int_0^{\infty} \hat{f}(\sigma_w) e^{-U_{de}^2 \Big/ 2\sigma_w^2 \left(\frac{\overline{A}}{\overline{C}}\right)^2} d\sigma_w \tag{13}$$

From equations (10) and (13), the distributions $\hat{f}(\sigma_w)$ and $\overline{G}(U_{de})$ are related by

$$\int_0^{\infty} \hat{f}(\sigma_w) e^{-U_{de}^2 \Big/ 2\sigma_w^2 \left(\frac{\overline{A}}{\overline{C}}\right)^2} d\sigma_w = \frac{V}{10560 N_o} \left(20P_1 e^{-U_{de}/2.2k_1} + 15P_2 e^{-U_{de}/5.3k_2}\right) \tag{14}$$

where, as indicated earlier (eq. (9)),

$$\frac{V}{10560N_o} \approx \frac{1}{15} \text{ and } \frac{1}{20}$$

for the airplanes used in the gust measurements. Thus, to this approximation

$$\int_0^\infty \hat{f}(\sigma_w) e^{-U_{de}^2 \Big/ 2\sigma_w^2 \left(\frac{\overline{A}}{\overline{C}}\right)^2} d\sigma_w = P_1 e^{-U_{de} \big/ 2.2k_1} + P_2 e^{-U_{de} \big/ 5.3k_2} \tag{15}$$

The solution of equation (15) is given by

$$\left.\begin{aligned} \hat{f}(\sigma_w) &= P_1 \sqrt{\frac{2}{\pi}} \frac{1}{b_1} e^{-\sigma_w^2 \big/ 2b_1^2} + P_2 \sqrt{\frac{2}{\pi}} \frac{1}{b_2} e^{-\sigma_w^2 \big/ 2b_2^2} \\ \hat{f}(\sigma_w) &= P_1 \hat{f}_1(\sigma_w) + P_2 \hat{f}_2(\sigma_w) \end{aligned}\right\} \tag{16}$$

where

$$b_1 = 2.2 \frac{\overline{C}}{\overline{A}} k_1$$

$$b_2 = 5.3 \frac{\overline{C}}{\overline{A}} k_2$$

$$\hat{f}_1(\sigma_w) = \sqrt{\frac{2}{\pi}} \frac{1}{b_1} e^{-\sigma_w^2 \big/ 2b_1^2}$$

$$\hat{f}_2(\sigma_w) = \sqrt{\frac{2}{\pi}} \frac{1}{b_2} e^{-\sigma_w^2 \big/ 2b_2^2}$$

The determination of the values of b_1 and b_2 thus depends upon $\frac{\overline{C}}{\overline{A}}$ which is the ratio of the acceleration response to unit discrete gusts of the standard form (cosine shape) and the root-mean-square acceleration response to a random gust input $\sigma_w = 1$. This ratio must be established for the airplane involved in the gust-data collection program.

For the single-degree-of-freedom case, vertical motion only (which appears adequate for present purposes as indicated in ref. 2),

$$\overline{A} = \frac{\rho VSm}{2W} \sqrt{\frac{I(K,s)}{\pi}} \tag{17}$$

where the term $\sqrt{\frac{I(K,s)}{\pi}}$ is a gust-response factor depending on K, the airplane mass ratio, and on s, the ratio of wing chord to scale of turbulence L. (See refs. 2 and 10.) Thus, from equations (12) and (17),

$$\frac{\overline{C}}{\overline{A}} = \sqrt{\frac{\rho_o}{\rho}} \frac{K_g}{\sqrt{I(K,s)/\pi}} \tag{18}$$

For present purposes, this ratio was evaluated on the basis of a characteristic transport configuration as given in table V of reference 2 in order to determine values of b_1 and of the Northrop P-61C airplane (the airplane actually used in the Thunderstorm Project gust survey) for the determination of b_2. The same form of gust power spectrum as that in reference 2 was used as well as a value of the scale of turbulence L of 1,000 feet. The ratio $\frac{\overline{C}}{\overline{A}}$ varies with altitude and the actual values obtained are given in table I. The values of b_1 and b_2 for the various altitude brackets are also given in the table. The associated probability density and cumulative probability distributions $\hat{f}(\sigma_w)$ and $\hat{F}(\sigma_w)$ for the various altitude brackets are given in figure 7. The distributions of σ_w for the nonstorm turbulence $\hat{f}_1(\sigma_w)$ and $\hat{F}_1(\sigma_w)$ and the storm turbulence $\hat{f}_2(\sigma_w)$ and $\hat{F}_2(\sigma_w)$ are also given separately in figure 8 for each of the altitude brackets.

APPLICATION TO GUST-LOAD CALCULATIONS

In the preceding section, a simplified description of the turbulence at the various altitudes was derived in terms of the probability density distributions of the root-mean-square gust velocity. This distribution is given by

$$\hat{f}(\sigma_w) = P_1 \frac{1}{b_1}\sqrt{\frac{2}{\pi}}\, e^{-\sigma_w^2/2b_1^2} + P_2 \frac{1}{b_2}\sqrt{\frac{2}{\pi}}\, e^{-\sigma_w^2/2b_2^2} \tag{19}$$

in which the parameters P_1 and P_2 represent the proportion of flight time (or distance) in nonstorm and storm turbulence, respectively, and b_1 and b_2 represent scale-parameter values for the individual probability distributions of σ_w for the two types of turbulence. The values of P_1, P_2, b_1, and b_2 varied with altitude. In this section, the foregoing specification of the turbulence environment is applied to the problems of missile gust-load-history calculations.

Estimation of Severe Gust Loads

As indicated in equation (3), the gust response history for a given airplane under given conditions, exposed to a gust history consisting of a series of locally stationary Gaussian processes of common spectral form (as, for example, defined by eq. (19)), may in general be expressed as

$$\overline{G}(y) = G_o \int_0^\infty \hat{f}(\sigma_w)\, e^{-y^2/2\sigma_w^2\overline{A}^2}\, d\sigma_w \tag{20}$$

where

y response quantity of concern (load, bending moment, stress, and so forth)

G_o number of response peaks per mile of flight in rough air

$\overline{A} = \sigma_y/\sigma_w$ for the specified spectral form of the gust input and, as indicated in reference 2, need not be restricted to single-degree-of-freedom systems

Substituting equation (19) into equation (20) and integrating yields

$$\overline{G}(y) = P_1 G_o e^{-y/b_1\overline{A}} + P_2 G_o e^{-y/b_2\overline{A}} \tag{21}$$

The results given by equation (21) may be viewed as a description of the statistics of the peak values of y and represent averages for extended operations under the specified conditions. As such, they do not apply directly to a single missile flight but must be viewed as the overall response histories of a large number of missiles for the specified conditions.

Equation (21) must be applied separately to each significant segment of the flight plan since the turbulence parameters P_1, P_2, b_1, and b_2 vary with altitude and the missile parameters G_o and $\overline{A}$ may also be expected to vary widely with the flight segment. If several flight segments are significant, either the overall load history $\overline{G}(y)$ must be determined as a weighted average (weighted, perhaps best, by the flight distances in each segment) or the load histories for individual flight segments must be considered separately. In many practical cases, one or two flight segments only are gust critical. This condition simplifies matters appreciably and is considered in a subsequent section.

If the load history, as specified by equation (21), is examined, it is clear that a small but finite probability of exceeding large values of y exists no matter what values of y are chosen. In any case, it is therefore impossible to select a value which will never be exceeded. Instead, it is necessary to accept some tolerable risk level or some finite probability of exceeding a chosen value. The actual probability value chosen would presumably depend upon the particular missile, the consequences of a structural failure, and economic and military tactical considerations. The question of the choice of the probability value is beyond the scope of this paper, and consideration herein is restricted to the problem of determining the load value once the probability of exceedance is chosen.

Consider the case of a single missile flight involving a flight distance D_r. This flight may be viewed as yielding a sample of the random process $y(t)$ of distance D_r. The random process $y(t)$ has an average of one exceedance of a specified value y_L in $\overline{D}(y_L)$ flight miles where

$$\overline{D}(y_L) = \frac{1}{\overline{G}(y_L)} \tag{22}$$

and where $\overline{G}(y)$ specifies the average load history of the missiles for extended flights. If it is assumed that the exceedances of y_L are distributed at random, then the probability of exceeding y_L in a given flight distance D_r is approximately given by

$$P_{ex}(y_L) \approx \frac{D_r}{\overline{D}(y_L)} \tag{23}$$

provided that $\overline{D}(y_L) \gg D_r$ which is assumed to be the case of interest. (The assumption of random distributions of y_L on $y(t)$ does not apply in a strict sense to the random process $y(t)$ as specified by the nonstationary input of equation (19). This assumption and the approximation of equation (23) are adequate for present purposes and are conservative to the extent that the cases of multiple values of y_L separated by a flight distance less than D_r are excluded.)

For given values of D_r and $P_{ex}(y_L)$, equations (22) and (23) specify the value of $\overline{G}(y_L)$. The result of the load calculation given by equation (21) may then be used to determine the required value of y_L to achieve the desired $P_{ex}(y_L)$. If several flight segments are being evaluated separately, the value of y_L may be determined in such a manner that the desired exceedance rate $P_{ex}(y_L)$ is given by

$$P_{ex}(y_L) = \sum \left[P_{ex}(y_L)\right]_i \tag{24}$$

where $\left[P_{ex}(y_L)\right]_i$ is the exceedance probability for the individual flight segments and the probabilities in the various segments are assumed independent.

A Simple Formula for Estimating Severe Gust Loads

In many cases of interest only a portion of the flight path or a single flight segment may be gust critical. If only a single flight segment is gust critical, it appears possible to derive a relatively simple formula for y_L in terms of a few significant quantities. For this purpose, it is of interest to examine the relative contributions to $\bar{G}(y)$ of the two terms on the right-hand side of equation (21). Let

$$\frac{\bar{G}(y)}{G_o} = \bar{G}_1(y) + \bar{G}_2(y) \tag{25}$$

where

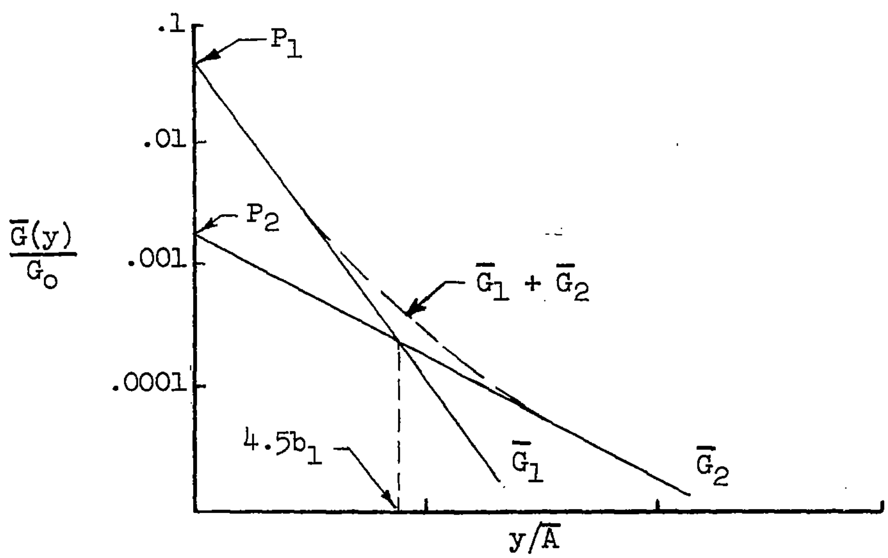
$$\bar{G}_1(y) = P_1 e^{-y/b_1\bar{A}}$$

$$\bar{G}_2(y) = P_2 e^{-y/b_2\bar{A}}$$

The gust data presented earlier indicate that for the significant altitude brackets

$$\left.\begin{aligned} P_1 &\approx 20P_2 \\ &, \\ b_2 &\approx 3b_1 \end{aligned}\right\} \tag{26}$$

For these conditions, the relative contributions of the two terms are schematically illustrated by the following sketch (a logarithmic scale applies to the ordinate):



As can be seen from the sketch, the principal contribution to $\overline{G}(y)$ arises from $\overline{G}_1(y)$ (the nonstorm-turbulence contribution) at low values of $\frac{y}{\overline{A}}$ and from $\overline{G}_2(y)$ (the storm-turbulence contribution) at high values of $\frac{y}{\overline{A}}$. Which of these two cases is of concern would appear to depend, in large part, on the particular missile and the desired exceedance rate. It is believed that the region of high values of $\frac{y}{\overline{A}}$ is of principal concern although, in some applications where operational considerations permit planning for the avoidance of storm turbulence, the $\overline{G}_1(y)$ case may alone be applicable.

In either case, equation (21) yields

$$\left.\begin{aligned} y_L &= b_i\overline{A}\left[\log_e P_i + \log_e G_o - \log_e \overline{G}\left(y_L\right)\right] \\ y_L &= b_i\overline{A}\left[\log_e P_i + \log_e G_o + \log_e \overline{D}\left(y_L\right)\right] \end{aligned}\right\} \qquad (i = 1,2) \tag{27}$$

Substituting for $\overline{D}\left(y_L\right)$ from equation (23) into equations (27) yields

$$y_L = b_i\overline{A}\left[\log_e P_i + \log_e G_o + \log_e \frac{D_r}{P_{ex}\left(y_L\right)}\right] \tag{28}$$

which is a simple and useful result. Equation (28) specifies a value of y_L in terms of the following groups of parameters:

(a) Gust input parameters b_i and P_i

(b) Missile response dynamics $\overline{A}$ and G_o

(c) Operational parameter D_r

(d) Desired exceedance rate P_{ex}

From figure 4 and table I, representative values of P and b for the altitude brackets of 0 to 40,000 feet are for the nonstorm-turbulence case

$$P_1 = 0.06 \qquad b_1 = 3.5$$

and for the storm-turbulence case

$$P_2 = 0.0025 \qquad b_2 = 10.5$$

Utilizing these values in equation (28) yields the following results:

For the nonstorm-turbulence case,

$$\left.\begin{aligned} y_L &= 3.5\bar{A}\left(\log_e 0.06 + \log_e G_o + \log_e \frac{D_r}{P_{ex}}\right) \\ y_L &= 3.5\bar{A}\left(\log_e G_o + E_1\right) \end{aligned}\right\} \tag{29}$$

where

$$E_1 = \log_e \frac{D_r}{P_{ex}} - 2.81$$

For the storm-turbulence case,

$$\left.\begin{aligned} y_L &= 10.5\bar{A}\left(\log_e 0.0025 + \log_e G_o + \log_e \frac{D_r}{P_{ex}}\right) \\ y_L &= 10.5\bar{A}\left(\log_e G_o + E_2\right) \end{aligned}\right\} \tag{30}$$

where

$$E_2 = \log_e \frac{D_r}{P_{ex}} - 6.0$$

The values of E_1 and E_2 are shown in figure 9 for a range of values of $P_{ex}(y_L)$ from 0.001 to 0.2 and for a range of values of D_r from 10 to 5,000 miles. The charts of figure 9 can be used directly along with the missile response parameters $\bar{A}$ and G_o to determine the load values in accordance with equations (29) and (30). The simple form of these results suggests that they could be used in preliminary design studies and in the development of design specifications.

In order to illustrate the applications of the foregoing results, an example is given. Consider a missile having a flight range in the lower atmosphere D_r of 100 miles and values for G_o of 10 and for $P_{ex}(y_L)$ of 0.01. For this case, a value of E_2 of 3.2 is obtained from figure 9(b). Using these values in equation (30) for the storm-turbulence case yields

$$y_L = 10.5\bar{A}\left(\log_e G_o + 3.2\right) = 58\bar{A} \tag{31}$$

It is of interest to note that doubling the range D_r to 200 miles yields

$$y_L = 64\bar{A}$$

or about a 10-percent increase in the value of y_L. $\Big($A 10-percent increase is also obtained if $P_{ex}(y_L)$ is reduced by one-half, that is, $P_{ex}(y_L) = 0.005.\Big)$

If the missile operations are restricted to the avoidance of storm areas and equation (29) for the nonstorm-turbulence case is considered applicable, the value obtained for the initial example is as follows:

$$y_L = 30\bar{A}$$

It is clear that a large reduction in the value of y_L (from $58\bar{A}$ to $30\bar{A}$) may be achieved by the avoidance of storm-turbulence areas. The structural penalty for all-weather missile operations thus appears large.

Estimation of Repeated Gust Loads

The problem of calculating the repeated loads and developing a fatigue loading differs in a significant respect from that of the limit load case. In the case of large loads, it is useful to consider the overall history of a fleet of missiles to insure that, on the average, the critical load is exceeded with a given frequency. In the fatigue case, the fleet concept in this form cannot be used. Instead, the cumulative load history of the individual missiles is of concern. The determination of such cumulative load histories requires information on the concurrent gust histories for the various flight segments of a particular missile flight. No information of this type is available. In some practical cases, significant simplifications may be feasible. One such possible simplification is considered herein.

It is assumed that the missile gust history for the significant part of the flight is statistically homogeneous and is specified by a given value of the root-mean-square gust velocity. This assumption may be expected to apply best to the case of missiles of short flight duration and appears, in general, to be conservative. On this basis, the cumulative load history for a given missile may be obtained from the following equation:

$$\overline{G}_t(y) = \sum D_i\overline{G}_i(y) \tag{32}$$

where

$\overline{G}_t(y)$ expected number of response peaks exceeding given values of y

D_i flight distance in ith flight segment

$\overline{G}_i(y)$ response history in ith segment which is obtained from

$$\overline{G}_i(y) = \left(G_o\right)_i e^{-y^2/2\sigma_w^2\overline{A}_i^2}$$

This procedure assumes that the root-mean-square gust velocity is constant but that G_o and $\overline{A}$ vary with flight segment. (It also assumes that the flight distance is sufficiently long to insure that the load history is close to the expected value $\overline{G}_t(y)$.) For a given missile operation, the load history (and thus the fatigue damage) from equation (32) depends only upon σ_w. The distribution of the load histories for a series of missiles, in turn, depends upon the probability distribution of σ_w. Thus, the specification of a value of σ_w which is exceeded with a given desired probability implies that the associated load history, as given by equation (32), is likewise exceeded with this same probability. For example, for a probability level of 0.001, figure 7(b) indicates that the value of σ_w exceeded with this probability varies between 6 and 11 for the various altitude brackets (ignoring the lowest altitude level). The conservative choice of a value for σ_w of

11 feet per second for calculations of repeated loads in equation (32) would thus yield a load history which would be exceeded with a probability of less than 0.001.

COMMENTS ON APPLICATIONS AND LIMITATIONS

The applications of the results obtained in the previous section to load calculations pose a number of problems. The applications, in general, require the determination of the quantities $\bar{A}$, G_o, D_r, and P_{ex}. The choice of values for the last quantity P_{ex} depends upon the particular problem and need not be of concern herein. The remaining quantities $\bar{A}$ and G_o, which define the missile response characteristics, and D_r, which depends upon the operational flight path, warrant some comment.

The quantities $\bar{A}$ and G_o, in practice, probably have to be determined by analytic means although, in some cases, direct experimental determinations may be possible. Analytically, these quantities may be defined as follows (ref. 2):

$$\bar{A} = \frac{1}{\sigma_w}\left[\int_0^\infty \Phi_w(\Omega)\left|H(\Omega)\right|^2 d\Omega\right]^{1/2} \tag{33}$$

$$G_o = \frac{5280}{\pi\sigma_y}\left[\int_0^\infty \Omega^2\Phi_w(\Omega)\left|H(\Omega)\right|^2 d\Omega\right]^{1/2} \tag{34}$$

where

$\Phi_w(\Omega)$ power spectrum of gust velocity

$H(\Omega)$ frequency-response function of missile, defining specified response of missile to unit sinusoidal gusts of frequency Ω

As specified by equations (33) and (34), no limitations exist, other than the usual one of a linear system, in the determination of $H(\Omega)$. Thus, in addition to the translational and rotational degrees of freedom, the effects of the missile control system and structural dynamics may be included in the analysis.

The determination of the appropriate value of D_r for a given missile operation may, in practice, also involve some difficulty. As utilized in equations (29) and (30), D_r is the flight distance in the gust-critical flight section. Inasmuch as the turbulence decreases rapidly above 40,000 to 50,000 feet, an upper limit in the value of D_r is the total flight distance below, say, 50,000 feet. In addition, only a small part of this flight distance may be at relatively high dynamic pressure. As a consequence, some arbitrary criterion for the determination of D_r, such as the flight distance below 50,000 feet and within 20 percent of the maximum dynamic pressure, may be desirable.

Although principal consideration has been given herein to the problems of gust-load calculations for missiles, the present results may also find application to other problems, such as the estimation of missile-motion response histories which may be required in guidance and tracking studies. In addition, many of the present results can, with minor modifications, be applied to airplane operations. For example, for transport-airplane operations (without radar for storm-turbulence avoidance) the appropriate value of P_2 in equation (30) differs from the value used for the all-weather missile case and instead would be based on the values given for airplane operation in figure 4.

The foregoing analysis based on turbulence data collected by airplanes in horizontal flight applies best to the case of missiles in flight paths similar to those of airplanes - that is, flight operations involving horizontal or moderately inclined flight paths. However, a large number of missile missions require rapid exit and entry through the lower atmospheric layers where air-motion disturbances are likely to give rise to significant loads. Missiles in such flight operations are likely to have near-vertical flight paths. For these cases, the use of airplane data is open to question for several reasons. First, the assumption of even local isotropy is probably most closely approximated in the atmosphere for horizontal layers and is unlikely to apply very well to the case of vertical flight paths because of the rapid changes in mean wind flow with altitude. This is particularly evident when it is recalled that turbulent areas are normally layers with a horizontal extent of 10 to 100 miles and with relatively thin vertical thicknesses of only several thousand feet. In addition, the rapid variations in horizontal wind speed with height (sometimes reaching values of 100 miles per hour in a few thousand feet as in jet-stream areas) are of an order of magnitude larger than the vertical gust velocities encountered in horizontal flight. These large wind shears exist at altitudes of 5 to 10 miles and appear to be the principal source of atmospheric disturbances applicable to missiles in vertical flight. In addition to those difficulties, missiles in vertical flight normally undergo such rapid variations in airspeed, dynamic pressure, and air density that it is questionable whether a locally time-invariant-system approach, as utilized herein, would apply. For these reasons, it is felt that the case of missile operations in near-vertical flight paths requires a separate and different approach centered upon direct measurements of the variations in horizontal wind with altitude as distinct from the measurements of turbulence obtained from airplanes in horizontal flight.

CONCLUDING REMARKS

Airplane measurements of atmospheric turbulence have been utilized to derive a simplified description of the atmospheric turbulence environment appropriate for missile operations. This description was then applied in developing an approach to the estimation of severe and repeated gust loads. Relations are given for calculating severe loads that are exceeded with a given probability as a function of turbulence parameters, the missile response characteristics, and the flight distance. Results are given for two cases: one which might be considered an all-weather operation and the other a limited-weather operation involving the avoidance of storm-turbulence areas. The levels of load values obtained for the two cases differ by a large amount. A simple procedure for estimating the repeated gust-load histories for missiles is also given.

Inasmuch as the present results are based on airplane measurements obtained in essentially horizontal flight, they appear applicable to missile flight operations involving only horizontal or near-horizontal flight. They do not, in particular, appear well suited for missile operations involving near-vertical flight paths through the lower atmosphere. For such operations, the changes in the horizontal wind with altitude appear to be the largest source of air-motion disturbance. This case appears to require a separate and different approach and one based on direct and detailed wind-shear measurements.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 16, 1958.

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TABLE I.- EVALUATION OF THE SCALE PARAMETERS b_1 AND b_2 IN GUST PROBABILITY DISTRIBUTIONS

(a) Scale parameter b_1

[$\bar{W}$ = 77,000 lb; S = 1,463 sq ft; $\bar{c}$ = 13.7 ft; m = 4.95 per radian; L = 1,000 ft]

Altitude, ft	$2.2k_1$	μ_g	K	K_g	$\sqrt{\frac{I(K,s)}{\pi}}$	$K_g \Big/ \sqrt{\frac{I(K,s)}{\pi}}$	$\sqrt{\rho_o/\rho}$	$\frac{\bar{C}}{\bar{A}}$	b_1
0 to 2,000	2.6	21.0	65.8	0.703	0.425	1.65	1.015	1.77	4.6
2,000 to 10,000	2.2	24.3	76.5	.722	.450	1.60	1.094	1.76	3.8
10,000 to 20,000	2.0	32.3	101.8	.758	.515	1.47	1.261	1.86	3.7
20,000 to 30,000	1.7	45.4	143.0	.798	.586	1.36	1.494	2.03	3.5
30,000 to 40,000	1.5	65.5	206.4	.823	.660	1.25	1.797	2.24	3.4
40,000 to 50,000	1.2	105.5	332.0	.840	.738	1.14	2.278	2.59	3.1
50,000 to 60,000	.9	170.0	535.0	.853	.800	1.07	2.893	3.08	2.8

(b) Scale parameter b_2

[$\bar{W}$ = 30,000 lb; S = 662.4 sq ft; $\bar{c}$ = 10.5 ft; m = 4.83 per radian; L = 1,000 ft]

Altitude, ft	$5.3k_2$	μ_g	K	K_g	$\sqrt{\frac{I(K,s)}{\pi}}$	$K_g \Big/ \sqrt{\frac{I(K,s)}{\pi}}$	$\sqrt{\rho_o/\rho}$	$\frac{\bar{C}}{\bar{A}}$	b_2
0 to 2,000	5.3	24.0	72.9	0.718	0.409	1.76	1.015	1.78	9.4
2,000 to 10,000	5.3	27.9	85.9	.736	.435	1.69	1.094	1.85	9.8
10,000 to 20,000	5.3	37.1	114.0	.768	.492	1.56	1.261	1.97	10.4
20,000 to 30,000	5.3	52.0	160.1	.798	.561	1.42	1.494	2.12	11.2
30,000 to 40,000	4.8	75.4	231.7	.824	.636	1.29	1.797	2.32	11.1
40,000 to 50,000	4.4	121.0	372.3	.840	.722	1.16	2.278	2.65	11.7
50,000 to 60,000	4.0	195.4	600.0	.856	.792	1.08	2.893	3.12	12.5

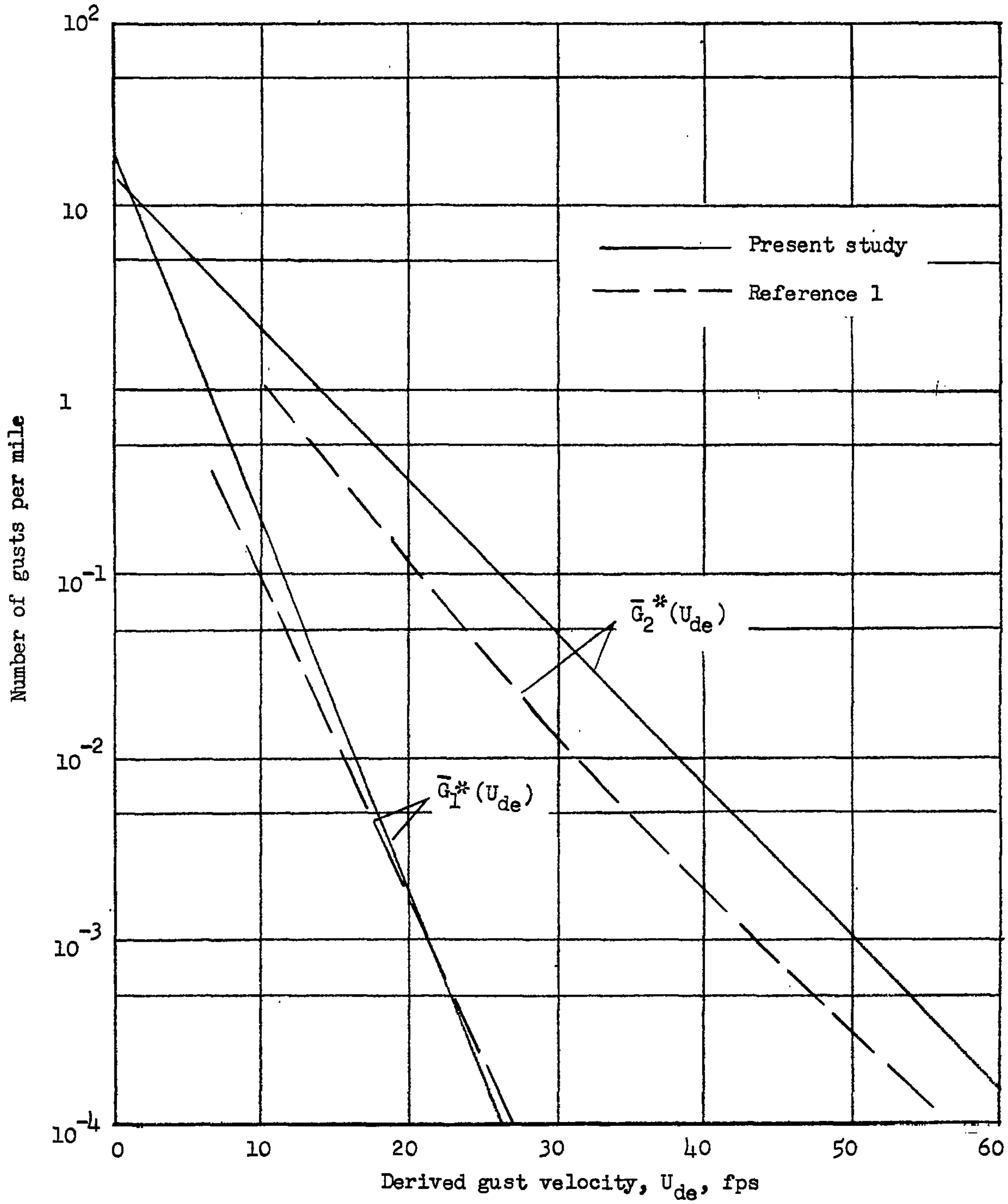


Figure 1.- Basic distributions of derived gust velocity $\bar{G}^*(U_{de})$ for storm- and nonstorm-turbulence conditions.

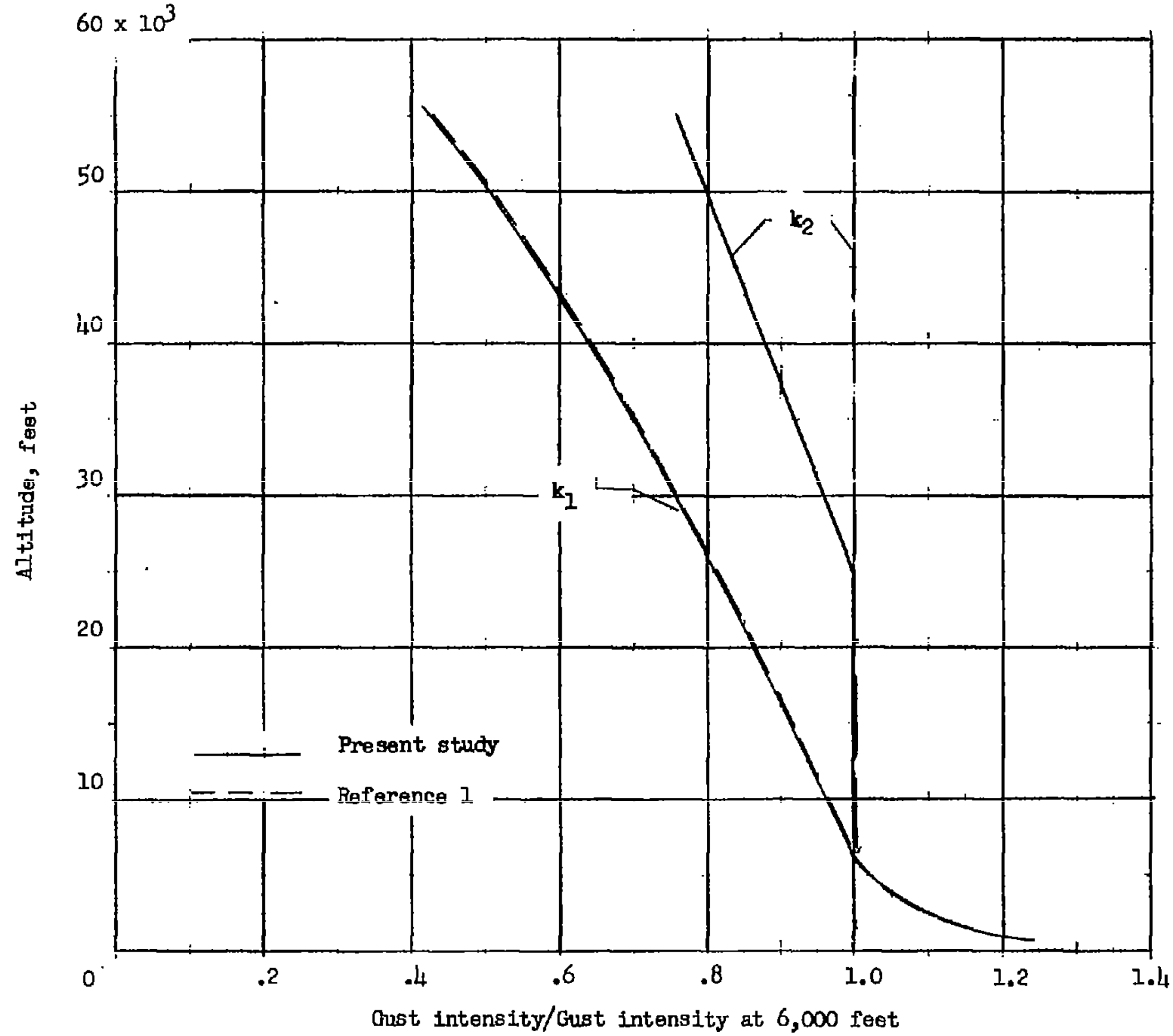


Figure 2.- Variation with altitude of turbulence intensity parameters k_1 and k_2.

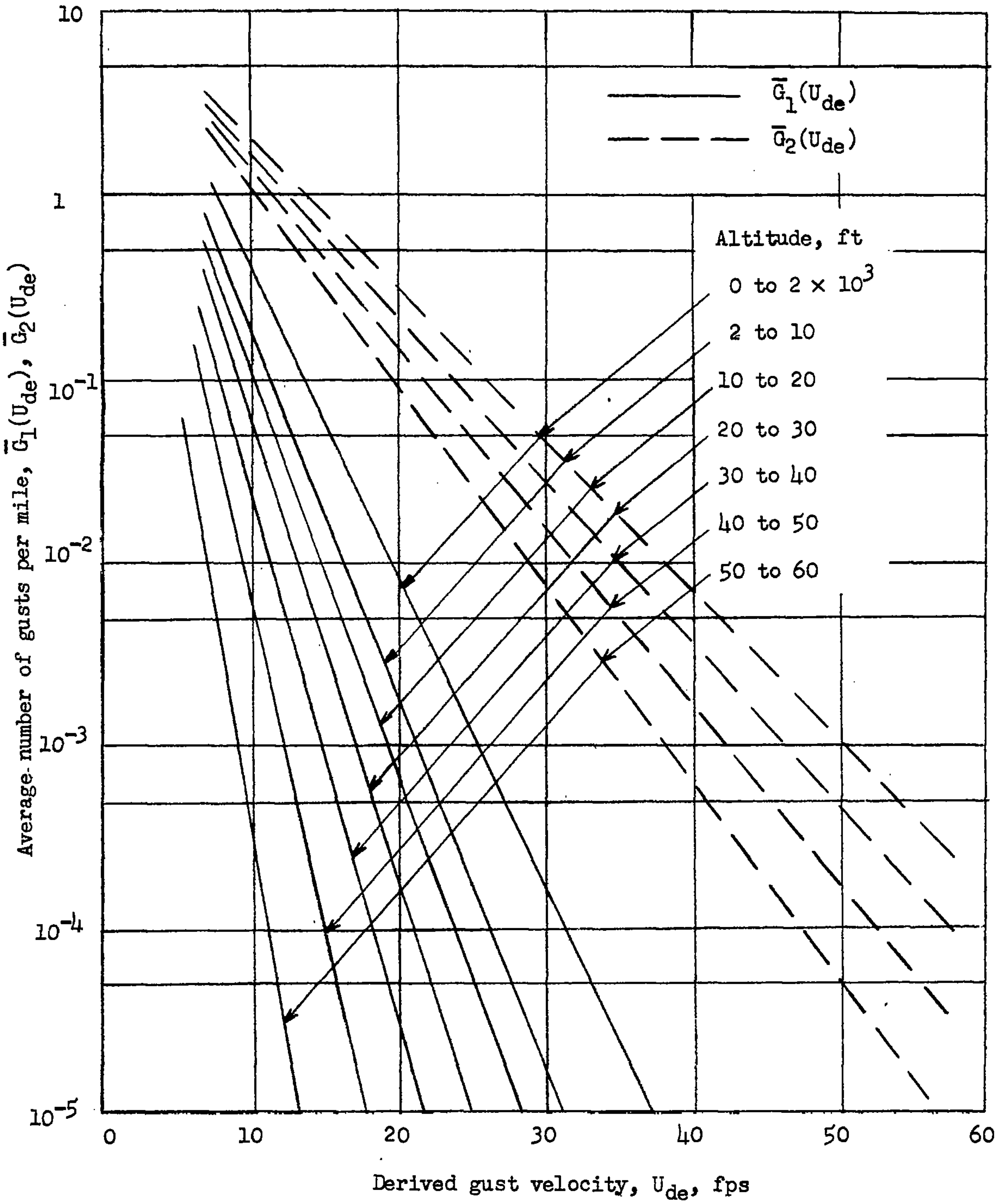
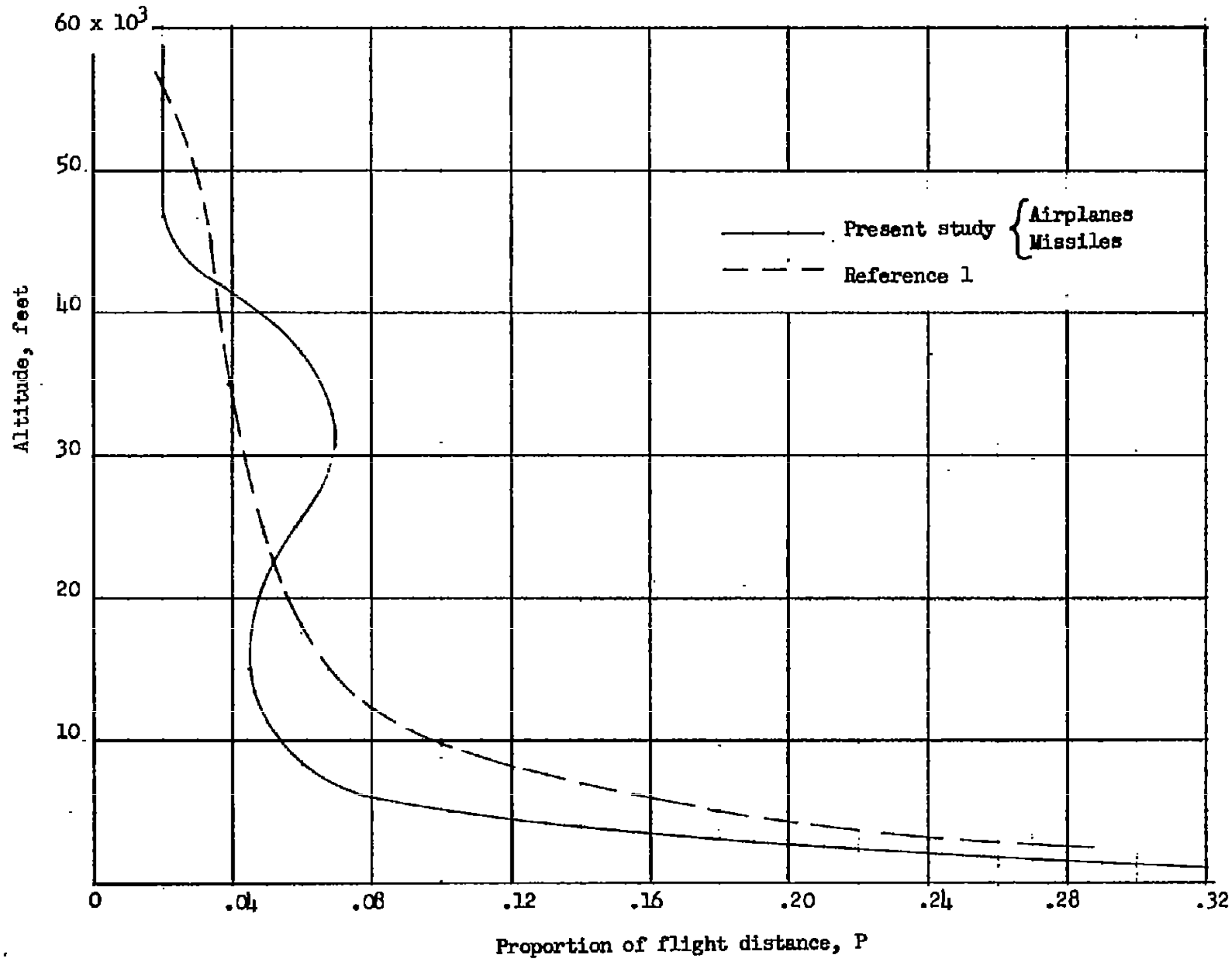
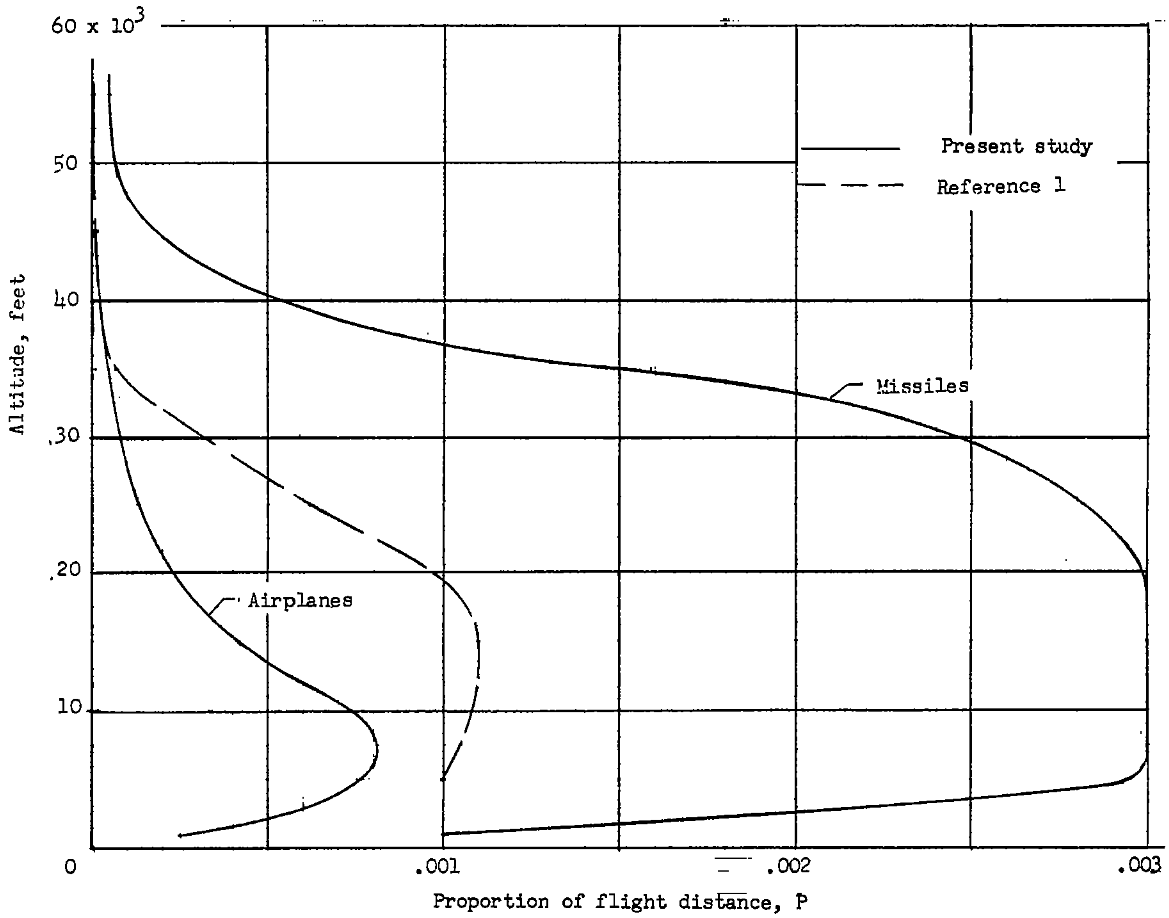


Figure 3.- Distributions of derived gust velocity for nonstorm turbulence $\bar{G}_1(U_{de})$ and storm turbulence $\bar{G}_2(U_{de})$ at various altitudes.



(a) Nonstorm turbulence.

Figure 4.- Proportion of flight distance in nonstorm and storm turbulence for airplanes and missiles.



(b) Storm turbulence.

Figure 4.- Concluded.

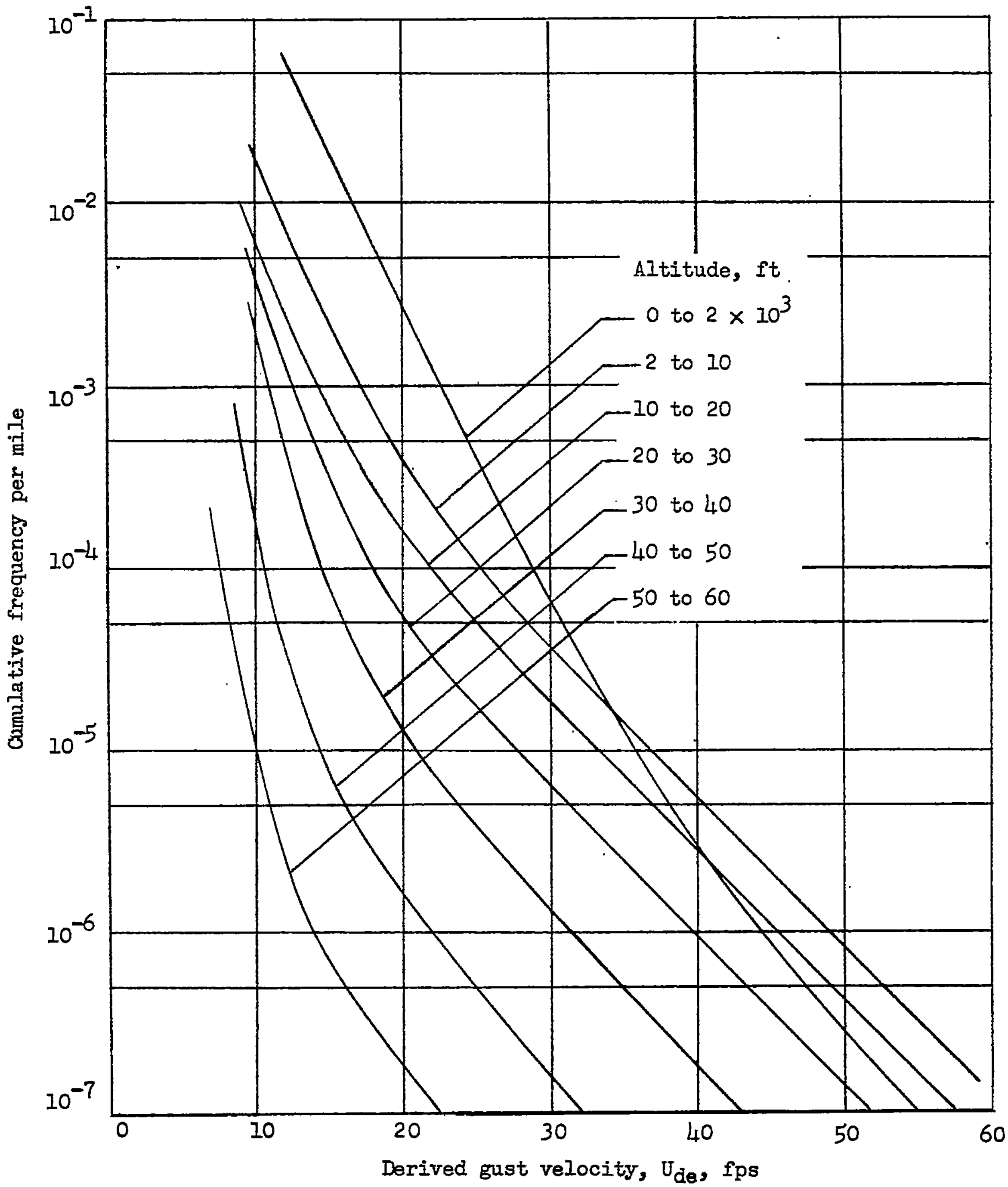


Figure 5.- Distribution of overall derived gust velocity $\overline{G}(U_{de})$ for airplanes at various altitudes.

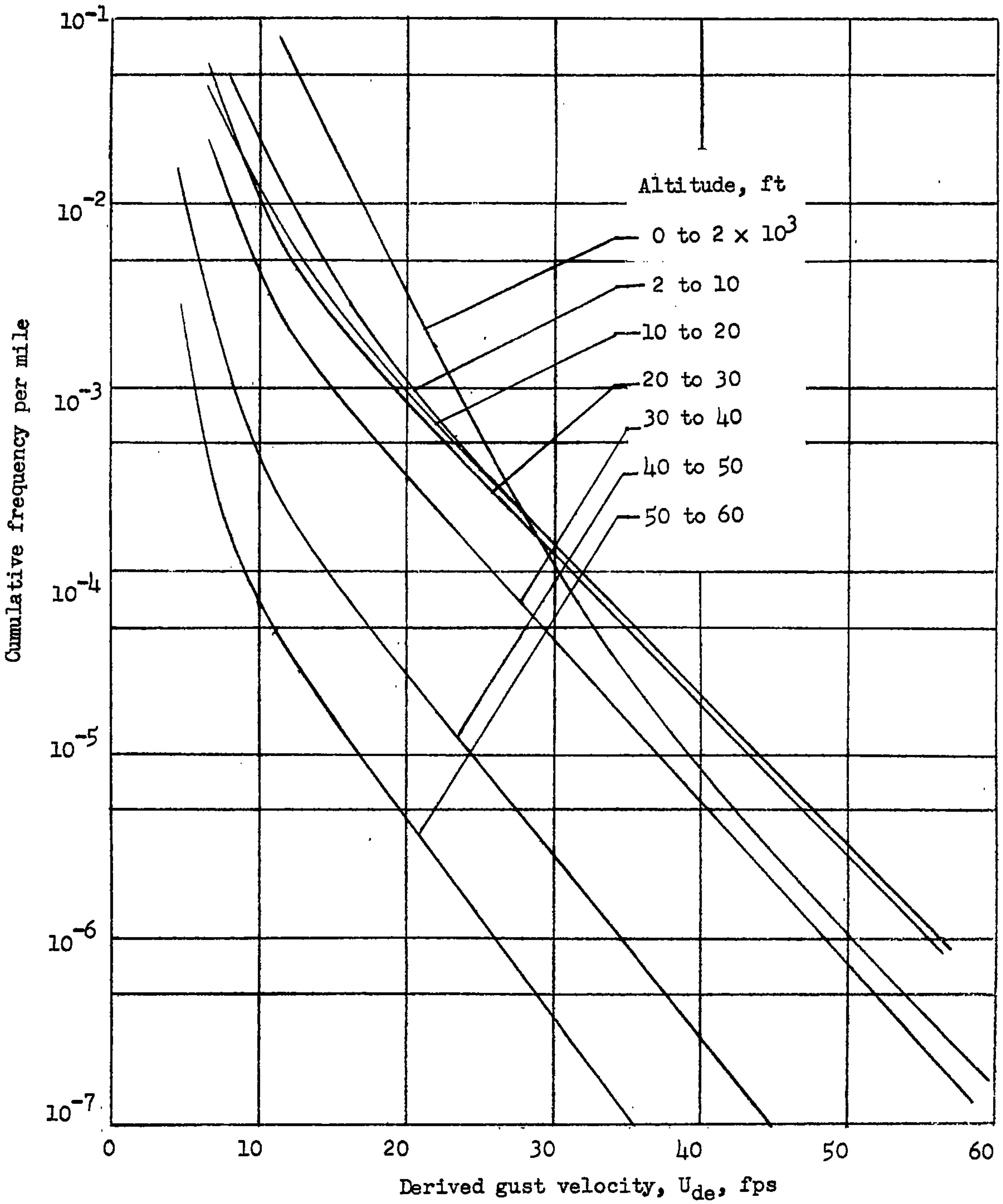
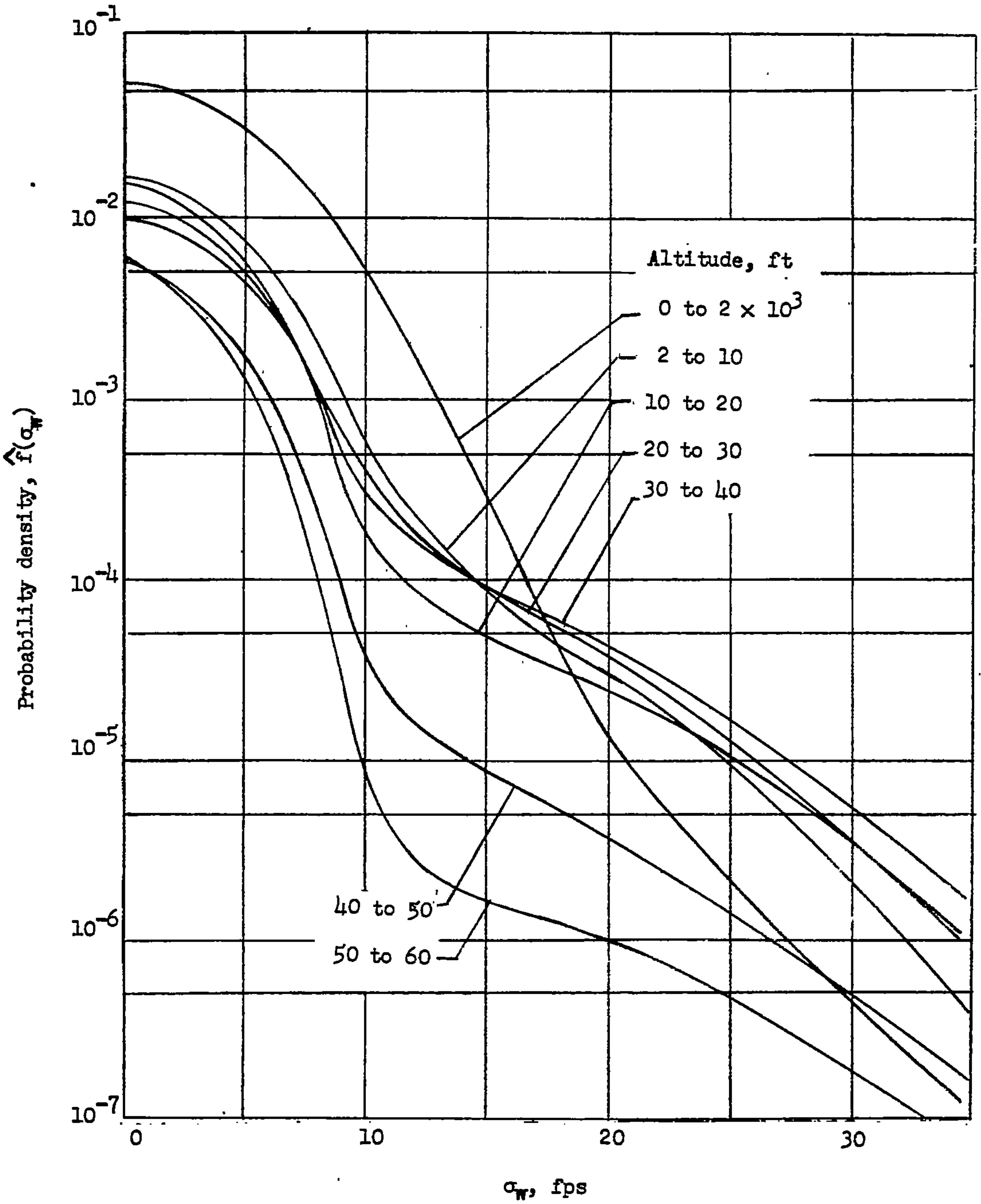
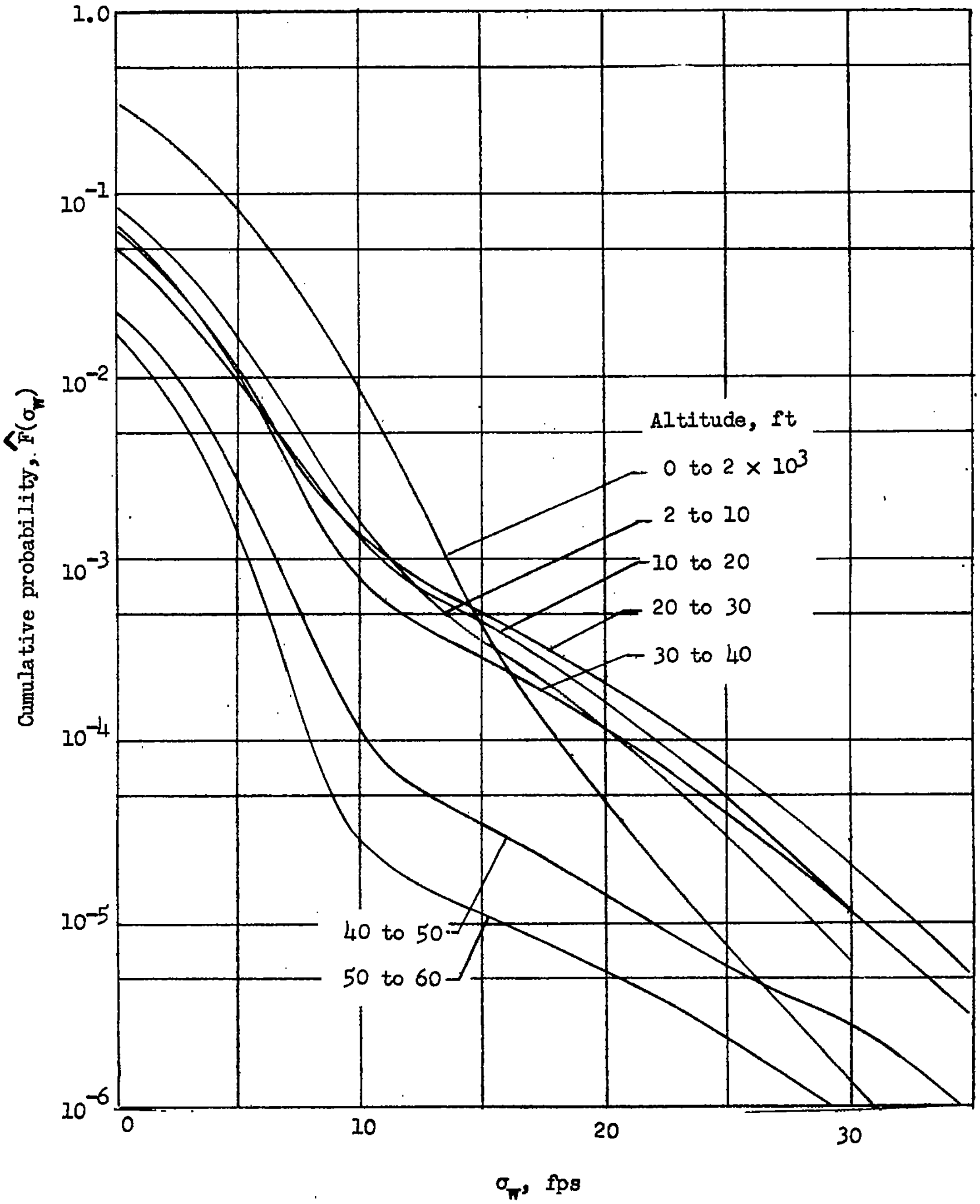


Figure 6.- Distribution of overall derived gust velocity $\overline{G}(U_{de})$ for missiles at various altitudes.



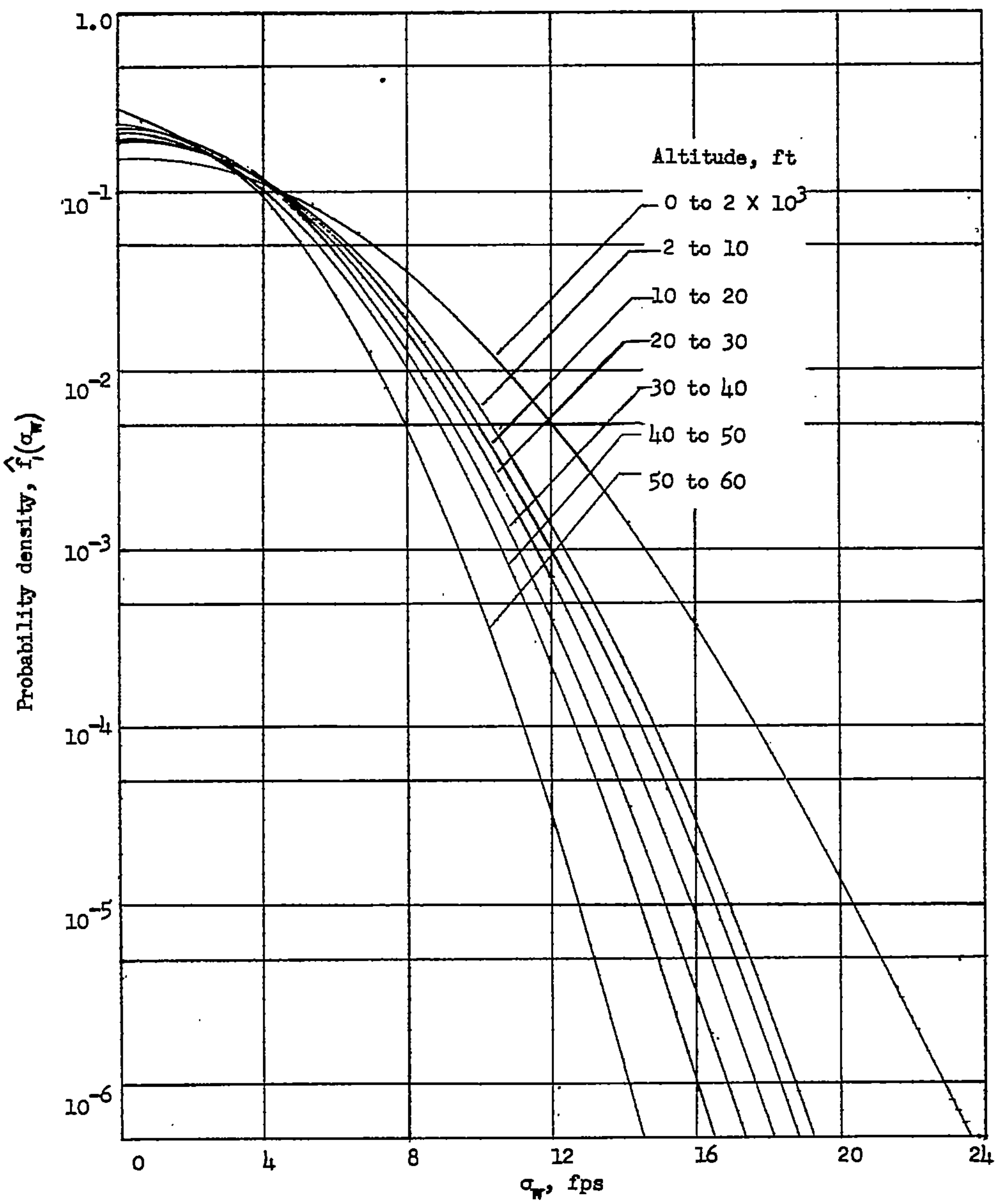
(a) Probability density.

Figure 7.- Probability density and cumulative probability distributions of root-mean-square gust velocity $\hat{f}(\sigma_W)$ and $\hat{F}(\sigma_W)$ for missiles at various altitudes.



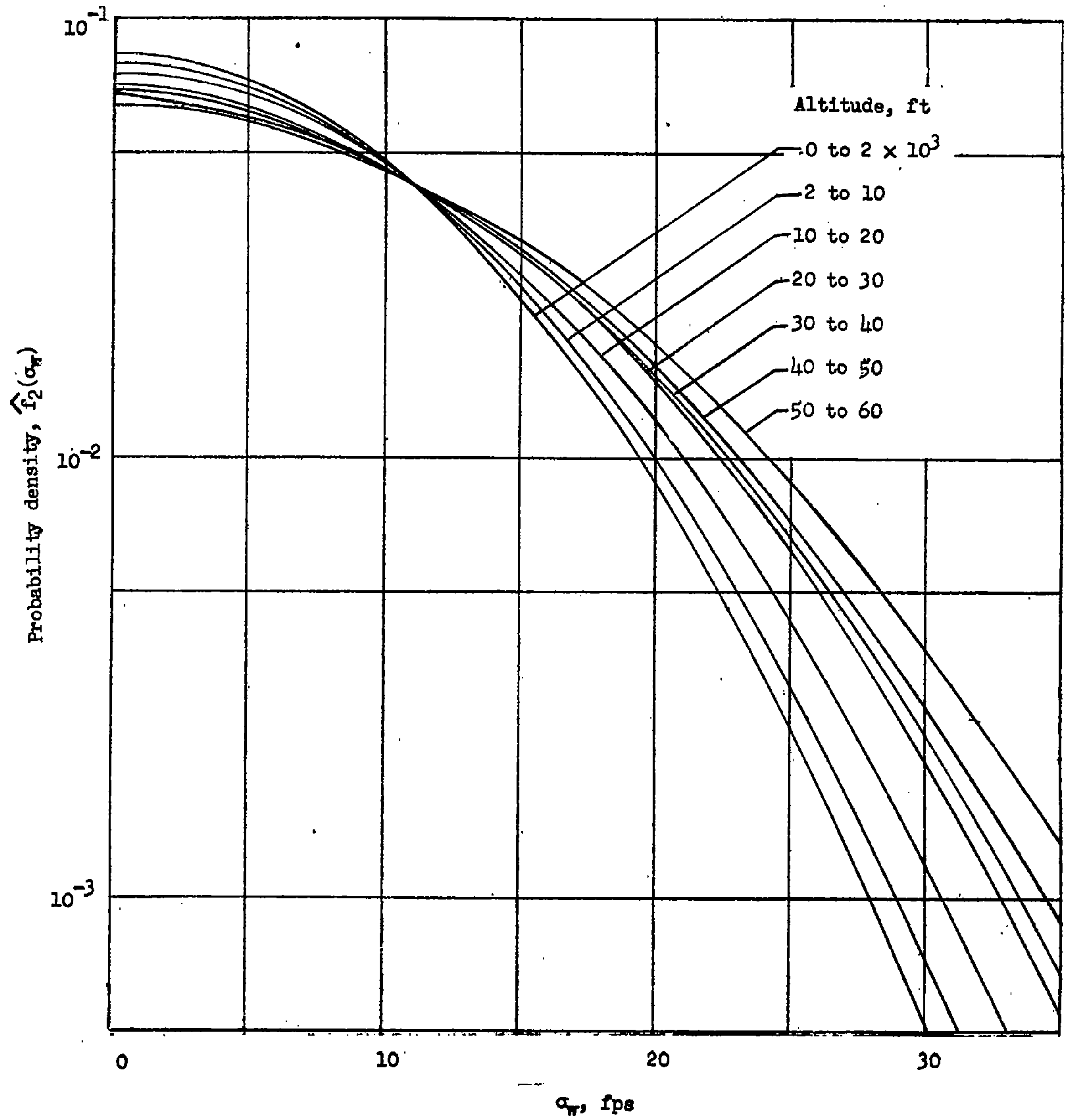
(b) Cumulative probability.

Figure 7.- Concluded.



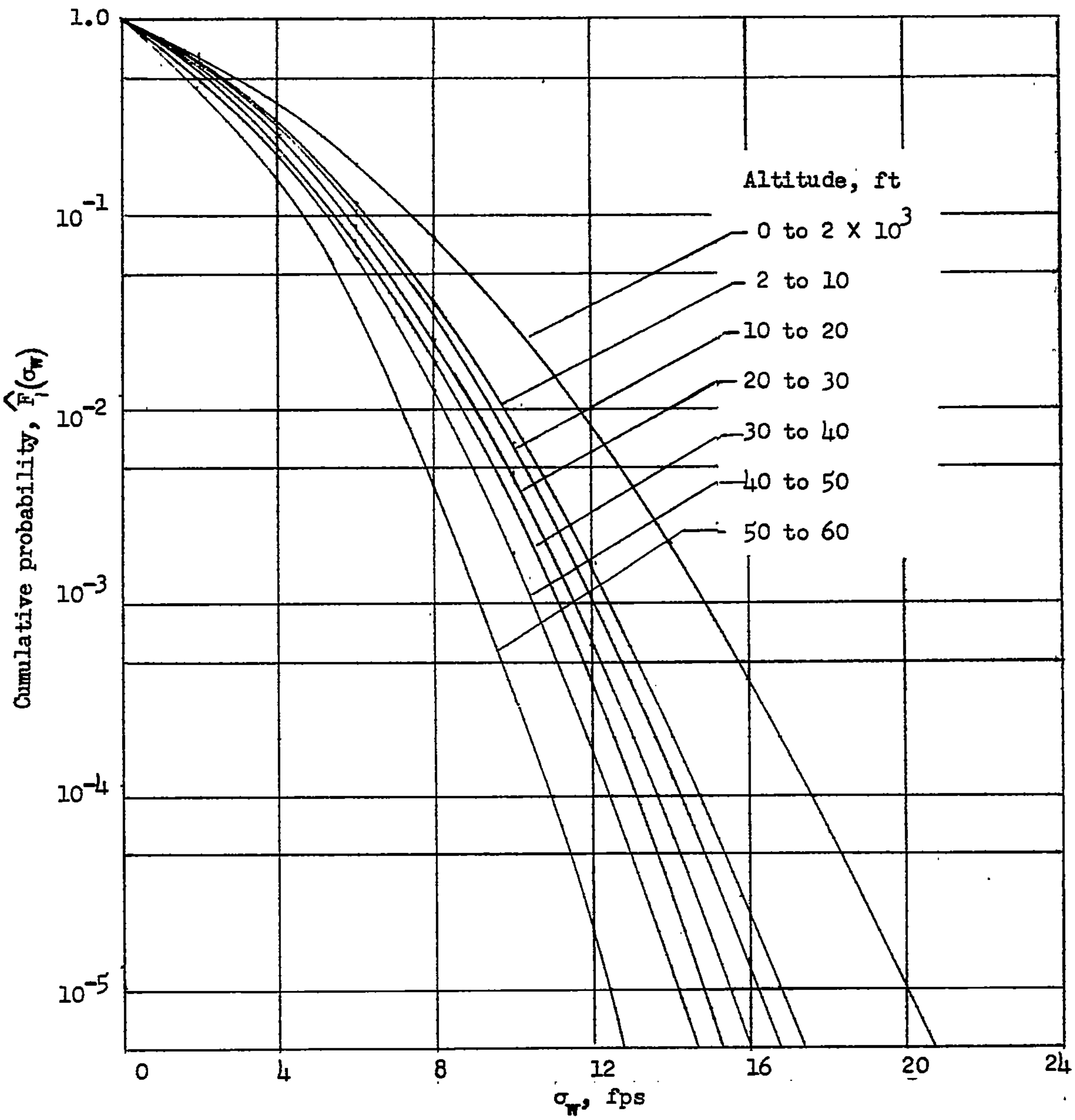
(a) Probability density for nonstorm turbulence $\hat{f}_1(\sigma_w)$.

Figure 8.- Probability density and cumulative probability distributions of root-mean-square gust velocity for nonstorm turbulence $\hat{f}_1(\sigma_w)$ and $\hat{F}_1(\sigma_w)$ and for storm turbulence $\hat{f}_2(\sigma_w)$ and $\hat{F}_2(\sigma_w)$ at various altitudes.



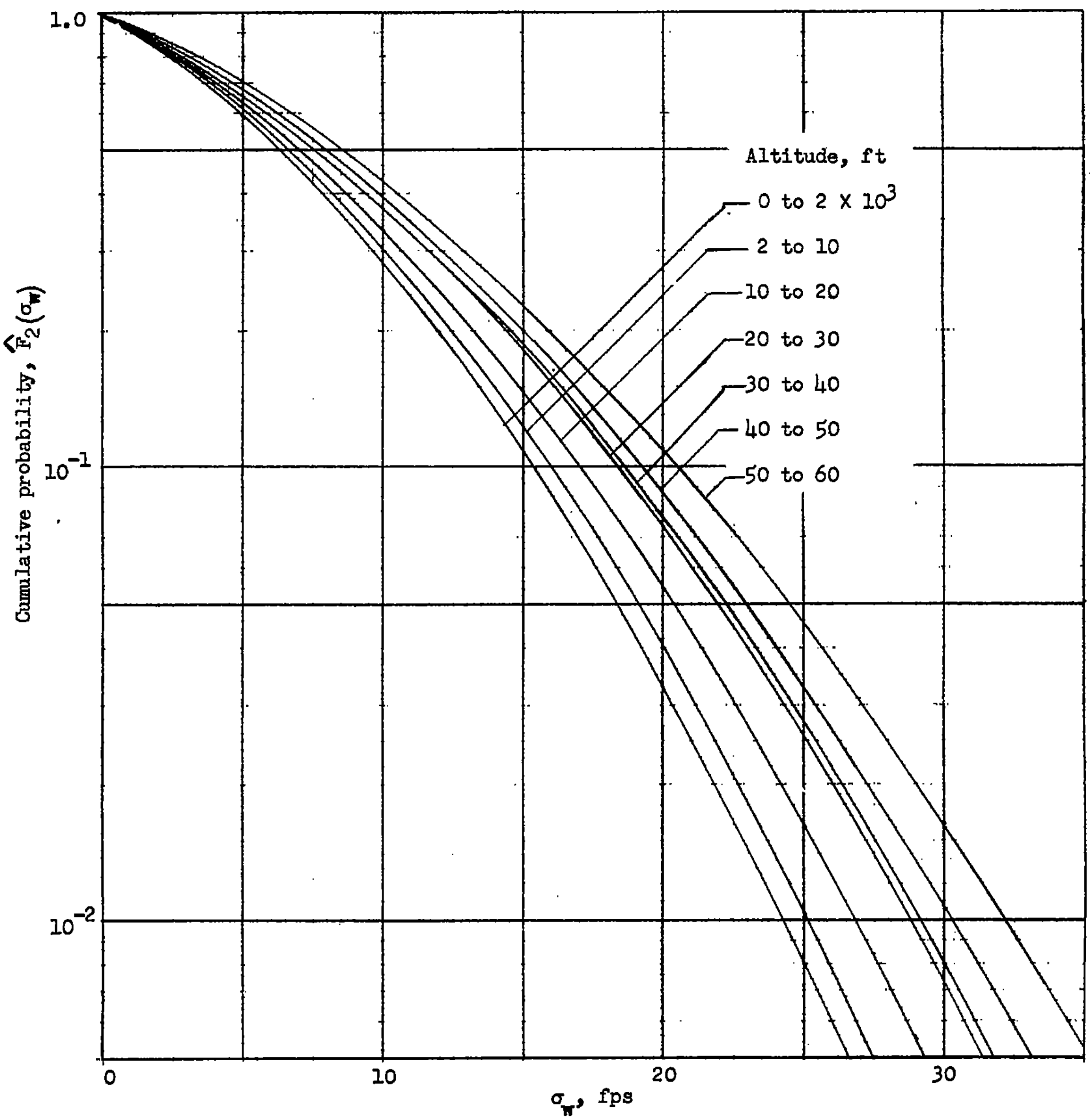
(b) Probability density for storm turbulence $\hat{f}_2(\sigma_w)$.

Figure 8.- Continued.



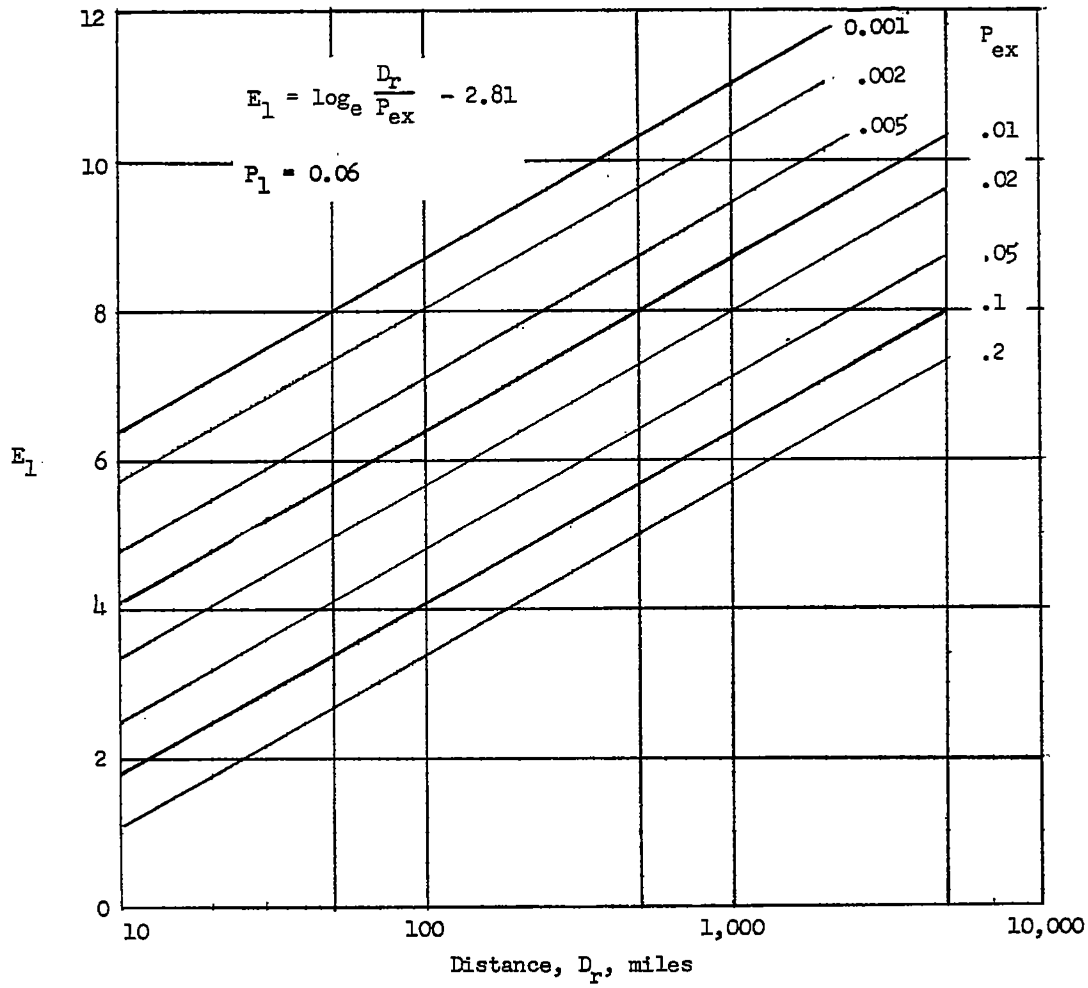
(c) Cumulative probability for nonstorm turbulence $\hat{F}_1(\sigma_w)$.

Figure 8.- Continued.



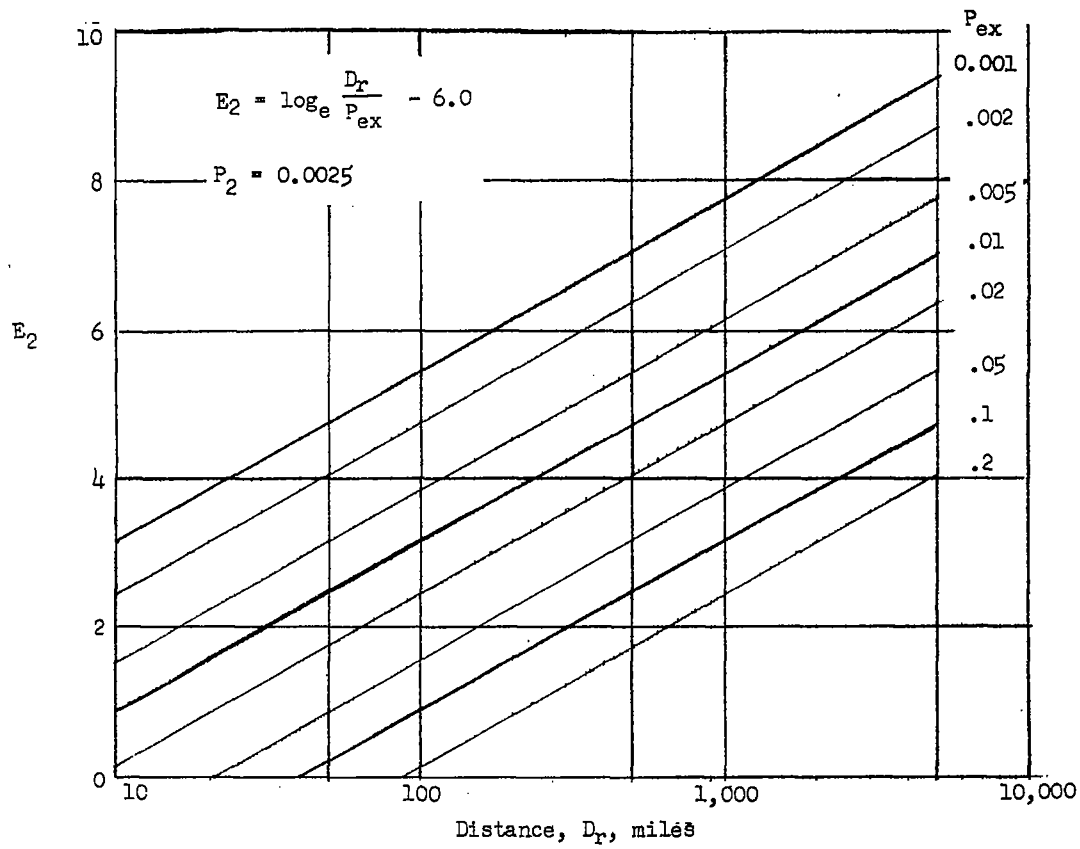
(d) Cumulative probability for storm turbulence $\hat{F}_2(\sigma_w)$.

Figure 8.- Concluded.



(a) Nonstorm-turbulence case E_1.

Figure 9.- Variation of E_1 and E_2 with D_r and P_{ex}.



(b) Storm-turbulence case E_2.

Figure 9.- Concluded.